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PROTON-MAGNETIC MONOPOLE BINDING ENERGY USING THE PAULI EQUATION

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Abstract

The binding energy of a magnetic monopole and proton are investigated using two distinct models. An unphysical Toy model, based on the Schrödinger equation and incorporating a nonconventional monopole, is the first approach. The second model utilizes a Dirac magnetic monopole. It includes a traditional Pauli equation approach and only physical interactions based on the proton spin, monopole magnetic field, and monopole charge. No additional *ad hoc* interactions are considered that would enhance the proton-monopole binding energy.

The Toy model yields a binding energy of 25 keV, and the more realistic Pauli equation model suggests that the proton-monopole system is only bound by 1.44 keV. Both Toy and Pauli equation binding energy results are weaker than most previous calculations.

KEYWORDS: Magnetic monopole, monopole-proton binding energy, Toy model, Pauli equation, realistic nonrelativistic interactions

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1. Introduction

A magnetic monopole is defined as a particle possessing an isolated magnetic charge that restores the electric and magnetic field symmetry in Maxwell's equations. Dirac's theory of magnetic monopoles [1-2] introduced these particles. Experimental monopole searches have yielded negative results [3-33] with the exception of a single experiment that has not been reproduced [34].

A Dirac magnetic monopole is a massive, fundamental point particle that may take any spin or mass. Dirac's theory of magnetic monopoles [1-2] offers an explanation for the quantization of electric charge.

Subsequent discussion and calculations preferentially use natural units ($\hbar$ = c = 1). SI units are only used for comparison to experiments to emphasize more commonly expressed results.

Dirac's quantization condition (in natural units) for magnetic monopoles predicts that the fundamental magnetic charge g_D of a monopole is a multiple of the fundamental electric charge

$$g_D = \frac{e}{2\alpha} \approx 68.5\, e \qquad (1)$$

where $\alpha \approx 1/137$ is the fine structure constant and e is the fundamental electric charge. The monopole charge can also exist as an integer multiple of g_D (i.e., ng_D where $n \ni \mathbb{Z}$).

In terms of its interaction with matter, magnetic monopoles and high-electric-charge objects (HECOs) (e.g., quark matter [35-36] and Q-balls [37 - 39]) have been studied. HECOs are analogous to monopoles in terms of their high charge, but not identical in terms of the associated physical interactions.

One of the most recent magnetic monopole and HECO searches [40] provided cross-section upper limits for spin-0 and spin-½ magnetic monopoles. These searches considered magnetic charge values of $1g_D$ and $2g_D$ and particles with $20 \leq |z| \leq 100$. [40] investigated masses between 200 and 4000 GeV/c^2. Although no magnetic monopoles or HECOs were detected, these results provide a useful framework for investigating the interaction of these particles with normal matter. These results support the contention that the monopole is significantly more massive than the proton.

This paper investigates the binding energy of a monopole and a proton. Previous models and results are summarized in Section 2, and the Toy and Pauli equation theoretical models are presented in Section 3. The numerical solution methodology is outlined in Section 4. New proton-monopole binding energy results are provided in Sections 5 and 6.

2.0 Previous Calculations

Previous calculations utilized both relativistic and nonrelativistic approaches. The calculations summarized in this section illustrate the diversity of approaches, and provide context for the present work. Effective Hamiltonians incorporate additional terms to increase the binding energy of a system. The Pauli model calculations summarized in Section 3.0 focus on strictly realistic physical interactions without introducing additional *ad hoc* terms.

2.1 Goebel

Goebel [41] examined monopole binding to a nucleus using the nonrelativistic Schrödinger equation. The associated Hamiltonian described the motion of a nucleus in the magnetic field of a monopole including its vector potential. A parameterized hard-core interaction was included in the model Hamiltonian. The resulting calculations predicted a binding energy 10 keV for the proton-monopole system.

2.2 Bracci and Fiorentini

Bracci and Fiorentini [42-43] considered a monopole with a magnetic charge bound to a nucleus utilizing a nonrelativistic Schrödinger equation. The associated interaction of the proton magnetic moment ($\vec{\mu}$) with the magnetic field $\vec{B}(r)$ generated by the monopole ($H_{int} = -\vec{\mu} \cdot \vec{B}(r)$) provided an attractive potential for a suitable spin orientation. Binding depends on the spin alignment of the proton with respect to the monopole's radial magnetic field. The aligned ($\theta = 0$) orientation produces a binding energy of 15.1 keV because the associated total interaction is attractive.

2.3 Makino, Maruyama and Miyamura

Makino, Maruyama and Miyamura [44] evaluated proton-monopole bound states using a non-relativistic effective Hamiltonian. The effective Hamiltonian included the effects on the proton by incorporating the nucleon form factor in the non-relativistic Hamiltonian. The resulting calculations yielded a proton-monopole ground state binding energy of 190 keV.

2.4 Olaussen et al.

Olaussen et al. [45] note that the proton-monopole system can only form bound states for the lowest angular momentum within a semi-relativistic formulation. A Kazama-Yang monopole-fermion model [45] was utilized in the

calculation to account for angular momentum coupling in the presence of a monopole.

The proton-monopole bound state utilized a modified Pauli-like Hamiltonian that included additional effects [45] . First, the model incorporated the fact that nucleons and nuclei have finite extensions. At short distances, the proton magnetic moment is modeled as a finite distribution. This accounts for the granular structure and the magnetic polarizability. Second, the model assumes that grand unification monopoles only carry a color magnetic charge. [45] notes that at short distances (inside the presumed screening radius) a strong interaction could be present. This finite extension leads to a correction to the Hamiltonian that is then related to a hadronic color charge distribution. [45] notes that these finite-size effects yield corrections to the Hamiltonian that have a $1/r^n$ dependence ($n > 2$) at large distances. These model assumptions lead to a proton-monopole binding energy of 263 keV.

2.5 Olaussen

Olaussen [46] determined the proton-monopole binding energy by considering large relative distances and nonrelativistic velocities utilizing a Pauli-like equation

$$\left[-\frac{1}{2m}\left(\vec{\nabla} - \mathrm{ieZ}\vec{A}\right)^2 - \vec{\mu}\cdot\vec{B}\right]\Psi = E\Psi \qquad (2)$$

where $\vec{A}$ is the vector potential of the magnetic monopole, m is the reduced mass of the monopole-nucleus system, and Ze is the charge of the nucleus. The solution of this equation yields two values for the proton-monopole binding energy (i.e., 236 ($\theta = \pi$) and 557 keV ($\theta = 0$)). These binding energies correspond to the two possible spin alignments of the proton relative to the monopole field.

2.6 Sivers

Sivers [47] used a nonrelativistic Schrödinger equation with a magnetic moment interaction. [47] controlled behavior of the wavefunction at short distances by incorporating a repulsive core with a radius comparable to that of the proton. The associated boundary condition quantized the wave function such that it vanished at the surface of the proton radius. Utilizing these assumptions, Sivers obtained a proton-monopole binding energy of 320 keV.

3.0 Model Calculations

This paper develops two models to determine the proton–monopole binding energy. The first is a crude, nonphysical Toy model that incorporates a nonconventional monopole. This model is analogous to a hydrogen-like system with the monopole replacing the proton, and the proton substituting for the electron. The Toy model is not a credible physical approach for determining the proton-monopole binding energy.

In the second model, a conventional Dirac monopole is included within the framework of the Pauli equation. This approach only utilizes realistic interactions. Short-distance corrections (e.g., finite extension, form factors, and hard cores) are included in some models [41-47]. However, no additional potential terms are added to the Pauli Hamiltonian proposed in this paper.

3.1 Toy Model

The Toy model Hamiltonian is based on the electrostatic interaction between the proton with charge e and mass m_p, and a massive magnetic monopole assumed to have a negative charge (Ze) with Z = -1. The corresponding binding energy is obtained by the solution of the nonrelativistic Schrödinger equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\ \frac{d\Psi}{dr}\right)+\frac{2m_p}{\hbar^2}\left[E_0+\frac{Z\,e^2}{r}-\frac{\hbar^2}{2m_p}\frac{l\,(l+1)}{r^2}\right]\Psi=0 \qquad (3)$$

In (3), r is the radial distance of the proton relative to the monopole located at the origin, l is the conventional orbital angular momentum, e is the unit charge, and Ψ is the spherically symmetric wavefunction.

3.2 Nonrelativistic Pauli Equation with Spin and Vector Potential

The second model utilizes a conventional point Dirac monopole within the framework of the Pauli equation. This approach yields a credible nonrelativistic model for the proton–magnetic monopole system. It is based on the Pauli Hamiltonian that includes the Dirac magnetic monopole vector potential and spin–magnetic field interaction.

The monopole is modeled as a classical spin-0 source of magnetic flux, consistent with the Dirac monopole model in its lowest-energy configuration. Possible effects of intrinsic monopole spin are not considered.

[48-53] document additional theoretical commentary relevant to the intrinsic characteristics of magnetic monopoles. In particular, [49,50,53] provides a comprehensive treatment of magnetic monopoles, and the associated basis for the calculations presented herein. In addition, [54-55] summarize additional information supporting the formulation of the Pauli Hamiltonian.

In developing the Pauli-equation framework for the proton–monopole system, several physical assumptions were adopted and include:

1. The monopole is modeled as a classical, spin-0 point source of magnetic flux without any internal structure. This choice follows the Dirac monopole formulation. No intrinsic monopole spin or spin–spin coupling with the proton is considered in the present analysis [49,50,53].

2. The monopole's magnetic field is derived from a classical treatment with a strength that is proportional to $\vec{r}/r^3$. Quantum fluctuations of the monopole field are neglected, consistent with the treatment of a heavy, static soliton.

3. The proton is treated as a point like spin-1/2 particle subject to the Pauli Hamiltonian, which incorporates both orbital motion and spin–magnetic coupling. Proton internal structure, finite size, and polarizability are not included in the model formulation.

4. The monopole mass is assumed to be much larger than the proton mass. Accordingly, monopole recoil is neglected. This reduces the system to a one-body problem in an external potential.

5. This work is primarily non-relativistic, employing the Pauli equation. Relativistic corrections (from the Dirac equation with monopole coupling) are not included.

6. The singularity associated with the monopole vector potential is treated in the standard gauge, where physical observables remain well-defined [49,50,53]. Possible effects of alternative gauge choices are not investigated in this paper.

The Dirac monopole is a point-like singular particle that has minimal magnetic charge (g) satisfying the Dirac quantization condition in natural units

$$g = \frac{n}{2e},\ n \in \mathbb{Z} \tag{4}$$

In this model, a spin 1/2 proton interacts with a spin 0 magnetic monopole that is defined by a vector potential. The monopole is fixed at the origin. The proton has mass m_p, charge +e, magnetic moment μ_p, and spin $\vec{S} = \vec{\sigma}\,\hbar/2$.

The monopole is characterized by a magnetic charge that produces a magnetic field

$$\nabla \cdot \vec{B} = 4\,\pi\, g\, \delta(r) \tag{5}$$

that has a singularity at the origin. This singularity is caused by the point monopole assumption. An extended monopoles would regularize this singularity.

A radial magnetic field is produced by the monopole [49,50,53]

$$\vec{B}(r) = g\,\frac{\hat{r}}{r^2} \tag{6}$$

The magnetic field and vector potential are related by the standard condition

$$\vec{B}(r) = \nabla \times \vec{A} \tag{7}$$

These defining relationships facilitate the specification of the Pauli Hamiltonian

$$H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 - \vec{\mu}_p \cdot \vec{B}(r) \tag{8}$$

where $\vec{p}$ is the conventional linear momentum. The proton magnetic moment (μ_p) is given by the relationship

$$\vec{\mu}_p = \frac{g_p\, e}{2m_p}\,\vec{\sigma} \tag{9}$$

where g_p is the proton gyromagnetic ratio ≈ 5.585.

To complete the specification of the Hamiltonian, the vector potential in spherical coordinates (in the singular Dirac gauge) is [49,50,53]

$$\vec{A} = \mathbf{g}\frac{(1-Cos(\theta)}{r\,Sin(\theta)}\,\hat{\phi} \tag{10}$$

The vector potential for the Dirac monopole is specified in the singular gauge along the z-axis. Care must be taken near θ=0 and π where the potential is singular. Calculations of angular momentum and kinetic energy use this potential consistently.

3.2.1 Total Angular Momentum with a Monopole

In the presence of a monopole, the canonical momentum $\left(\vec{p} - e\vec{A}\right)$ is not

conserved, because it includes the magnetic vector potential. In order to restore conservation of angular momentum, a generalized angular momentum (T) is defined [45,46,48-55]

$$T = \vec{r} \times \left(\vec{p} - e\vec{A}\right) - q\,\hat{r} \tag{11}$$

where q = e g is the dimensionless monopole charge number, and g is the unit magnetic monopole charge. q is defined from the Dirac quantization condition

$$q = \frac{n}{2},\ n \ni \mathbb{Z} \tag{12}$$

3.2.2 Basic Wavefunction for the Pauli Equation

Before specifying the Pauli equation for the proton-monopole system, the relevant wavefunction is defined. Assuming a separation of variables in spherical coordinates, the wavefunction is defined as [45,46,48-55]

$$\Psi(r) = u(r)\ \chi_s\, Y_{q,t,m}\ (\theta, \phi) \tag{13}$$

where u(r) is the radial wavefunction, $Y_{q,t,m}\ (\theta, \phi)$ are monopole harmonics [49-53] that are analogous to conventional spherical harmonics, and χ_s is a two component spinor. In (13), t is the generalized angular momentum quantum number in the presence of the monopole, and m is the generalized magnetic quantum number.

The introduction of the monopole charge number q complicates the angular momentum structure. For nonzero q values, generalized angular momentum and generalized total angular momentum ($\vec{F}$) are defined as

$$\vec{T} = \vec{L} - q\,\hat{r} \tag{14}$$

$$\vec{F} = \vec{T} + \vec{S} \tag{15}$$

where $\vec{L}$ is the conventional orbital angular momentum, q is the monopole charge number that shifts the minimum allowed angular momentum upward from 0 to $|q|$, and $\hat{r}$ is the unit radial vector. When q = 0, (14) and (15) revert to their conventional forms.

These generalized angular momentum operators yield the following quantum number equations for the monopole harmonics

$$T^2\, Y_{q,t,m}\,(\theta,\phi) \;=\; t(t+1) Y_{q,t,m}\,(\theta,\phi) \tag{16}$$

$$T_z\, Y_{q,t,m}\,(\theta,\phi) \;=\; m\, Y_{q,t,m}\,(\theta,\phi) \tag{17}$$

where

$$t \;=\; |q|, |q|+1, |q|+2, \ldots, \; |q|+l \tag{18}$$

$$m \;=\; -t, -t+1, \ldots, 0, \ldots, \; t-1, \; t \tag{19}$$

For comparison, the conventional angular momentum and its projection on the z axis (L_z) operates on spherical harmonics to yield the expected analogous results

$$\vec{L}^2\, Y_{l,m}\,(\theta,\phi) \;=\; l(l+1) Y_{l,m}\,(\theta,\phi) \tag{20}$$

$$L_z\, Y_{l,m}\,(\theta,\phi) \;=\; m\, Y_{l,m}\,(\theta,\phi) \tag{21}$$

The generalized total angular momentum F and its projection F_z yield the expected results

$$\vec{F}^2\, \Psi(r) \;=\; f(f+1) \Psi(r) \tag{22}$$

$$F_z\, \Psi(r) = \; m_f\, \Psi(r) \tag{23}$$

where

$$f \;=\; |t-s|, |t-s|+1, \; |t-s|+2, \ldots t+s-2, \; t+s-1, \; t+s \tag{24}$$

$$m_f \;=\; -f, -f+1, \; -f+2, \ldots, f-2, \; f-1, \; f \tag{25}$$

3.2.3 Proton-Monopole Hamiltonian Kinetic Energy (K) Term

In spherical coordinates the kinetic energy term in the presence of the vector potential [45,46,48-55] is

$$K \;=\; \frac{1}{2\, m_p} \left(\vec{p} - e\vec{A}\right)^2 = \; -\frac{\hbar^2}{2m_p} \left(\nabla \;-\; i\, \frac{e}{\hbar}\, \vec{A}\right)^2 \tag{26}$$

Expanding the Laplacian in spherical coordinates leads to an expression that involves a generalized angular momentum operator ($\bar{\Lambda}$)

$$K \;=\; -\frac{\hbar^2}{2m_p} \left[\frac{1}{r^2}\, \frac{\partial}{\partial r} \left(r^2\, \frac{\partial}{\partial r}\right) - \frac{1}{r^2}\, \widehat{\Lambda}^2\right] \tag{27}$$

and

$$\widehat{\Lambda}^2 = \left[\frac{1}{Sin\theta}\frac{\partial}{\partial\theta}\left(Sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{Sin^2\theta}\left(\frac{\partial}{\partial\phi} + iq\,Cos\theta\right)^2\right] \tag{28}$$

The kinetic energy in the presence of a monopole is represented by a generalized angular momentum operator, which acts on the monopole harmonics. This operator accounts for both orbital angular momentum and the monopole-induced shift, ensuring that the centrifugal term in the radial equation correctly incorporates the monopole effect. Physically, the generalized angular momentum operator in (28) modifies the centrifugal term due to the monopole's vector potential

$$\widehat{\Lambda}^2\; Y_{q,t,m}\;(\theta,\phi) = l_{eff}\left(l_{eff}+1\right) Y_{q,t,m}\;(\theta,\phi) \tag{29}$$

where l_{eff} is defined in (34).

Utilizing these results the kinetic energy portion of the Pauli equation becomes

$$\frac{1}{2m_p}\left(\vec{p} - e\vec{A}\right)^2 \Psi(r) = -\frac{\hbar^2}{2m_p}\left[\frac{d^2}{dr^2} - \frac{l_{eff}\left(l_{eff}+1\right)}{r^2}\right]\Psi(r) \tag{30}$$

3.2.4 Proton-Monopole Hamiltonian Spin-Magnetic Coupling (V_{spin}) Term

The spin-magnetic coupling term in the Pauli Hamiltonian [45,46,48-55] is defined as

$$V_{spin} = -\vec{\mu}_p \cdot \vec{B}(r) = -\frac{g_p\, e}{2m_p}\,\vec{\sigma}\cdot\vec{B} = -\frac{g_p\, e\, g}{2m_p\, r^2}\,\vec{\sigma}\cdot\hat{r} = -\frac{\alpha_s}{r^2}\,\vec{\sigma}\cdot\hat{r} \tag{31}$$

As will be illustrated below, the spin–magnetic interaction produces eigenvalues $\pm\alpha s / r^2$ depending on the proton spin alignment along the radial direction of the monopole magnetic field. Since this interaction term is critical for forming a bound state, it can overcome the centrifugal barrier if it is sufficiently attractive. This spin-magnetic interaction term is positive (negative) if the monopole magnetic field and the proton spin are aligned (anti-aligned).

Eigenvalues of the $\vec{\sigma}\cdot\hat{r}$ are derived from the action of this operator on the two component spinor. This is a standard result for helicity-like operators

[45,46,48-55]

$$(\vec{\sigma} \cdot \hat{r})\,\chi_{\pm} = \pm\,\chi_{\pm} \tag{32}$$

The + (-) sign correspond to the spin direction aligned (anti-aligned) along the radial direction. When V_{spin} acts on the two component spinor, the operator $-\vec{\sigma} \cdot \hat{r}$ returns the value $\mp \frac{\alpha s}{r^2}$.

3.2.5 Radial Pauli Equation

Combining these results leads to a radial Pauli Hamiltonian that only includes basic physical interactions and no *ad hoc* terms. The resulting radial equation follows from performing the operator actions associated with the proton spin, magnetic moment, and magnetic monopole field that were noted previously. The associated algebraic manipulations required to obtain the results of these evaluations are outlined in [45,46,48-53], and lead to the following simplified radial equation

$$-\frac{d^2u(r)}{dr^2} + \left[\frac{l_{eff}\,(l_{eff}+1)}{r^2} \mp \frac{\alpha_s}{r^2}\right] u(r) = 2\,m_p\,E\,u(r) \tag{33}$$

where the effective angular momentum [45,46,48-53] is

$$l_{eff}\,(l_{eff}+1) = t(t+1) - q^2 + sq \tag{34}$$

Inserting (34) into (35) yields

$$-\frac{d^2u(r)}{dr^2} + \frac{1}{r^2}[\,t(t+1) - q^2 + sq \mp \alpha s]\,u(r) = 2\,m_p\,E\,u(r) \tag{35}$$

where $\alpha = q/m_p$.

From a physical perspective, the first term in (35) represents the kinetic energy derived from the orbital motion. This term involves the generalized orbital angular momentum t, and the t(t + 1) term is the main centrifugal contribution to the Pauli Hamiltonian. For an ordinary central potential (q = 0), the centrifugal barrier arises from the conventional orbital angular momentum.

The second term ($-q^2$) is the angular momentum shift due to the presence of the monopole. The inclusion of the monopole causes L to no longer be a good quantum number. As a result of the monopole, the second term produces an effective reduction in the centrifugal barrier that facilitates binding of the proton and monopole.

The third term represents the proton spin correction. It is derived from the inherent spin of the proton and the magnitude of the magnetic charge.

The fourth term represents the coupling of the proton spin and radial monopole magnetic field. This term depends on the proton spin orientation relative to the monopole's radial magnetic field. Physically, the spin orientation shifts the effective angular momentum barrier and yields distinct energy levels for each spin orientation.

4.0 Solution Methodology for the Pauli Equation

To obtain a numerical solution for the proton-monopole binding energy, the spatial domain is modeled using a finite-difference method. The second-order spatial derivatives (e.g. the kinetic-energy term) are replaced by central-difference formulas evaluated on a spatial grid. This transformation converts the continuous differential equation into a matrix eigenvalue problem. For a one-dimensional system, or for radial equations obtained through separation of variables, the finite-difference approximation leads naturally to a symmetric tridiagonal matrix representation of the Hamiltonian [56,57].

The diagonal elements of the resulting matrix contain contributions from the kinetic-energy terms, magnetic interaction terms, and any other position-dependent contributions appearing in the Hamiltonian. The off-diagonal elements arise from the finite-difference approximation of the second derivative and represent coupling between adjacent grid points.

The numerical procedure then solves the matrix eigenvalue problem

$$H\Psi = E\Psi \tag{36}$$

where H denotes the Hamiltonian matrix, E an energy eigenvalue, and Ψ the corresponding wavefunction. Since the Hamiltonian is real and symmetric, under the assumptions employed in the present formulation, the matrix possesses a complete set of real eigenvalues and orthogonal eigenvectors [58].

Recursive relations are used to identify intervals containing eigenvalues, while iterative procedures progressively refine the estimates until the solution is obtained [58-60]. The final result of the numerical procedure is a solution of the Pauli Hamiltonian that yields the computed energy levels and the corresponding normalized eigenstates.

5.0 Summary of Binding Energy Results

Table 1 provides a summary of proton-monopole binding energy calculations from the literature and those presented in this paper. Explicit model assumptions were noted in Section 2 for previous calculations and Section 3 for the models presented in this paper. The calculations summarized in Section 2 included modifications to the Hamiltonian that strengthened the proton-monopole potential and enhanced the binding energy. No such modifications were made to the Pauli equation calculations in this paper. Section 6 provides additional discussion of the Toy and Pauli equation results.

Table 1 Summary of Proton-Monopole Binding Energy Calculations

Reference	**Model Basis**	Binding Energy (keV)
Goebel [41]	Schrödinger equation	10
Bracci and Fiorentini [42-43]	Schrödinger equation	15.1
Makino et al. [44]	Schrödinger equation	190
Olaussen et al. [45]	Pauli equation	263
Olaussen [46]	Pauli equation	236 and 557
Sivers [47]	Schrödinger equation	320
This work	Toy Model (Schrödinger equation)	25
This work	Pauli Equation	1.44

6.0 Results and Discussion

As noted in previous commentary and the calculations summarized in Table 1, the binding energy of a proton and magnetic monopole depends on the assumed structure of the Hamiltonian and theoretical model (e.g., Schrödinger and Pauli equations). Calculations summarized in [41-47] also demonstrate the impact of including the proton spin, monopole characteristics, and *ad hoc* factors on the proton-monopole binding energy. The assumed Dirac monopole provides an initial formulation, but other monopole formulations are possible [53].

6.1 Toy Model

The Toy model has very limited physical content and incorporates an unphysical monopole and a Coulomb interaction. This model does not provide an

accurate depiction of the physical proton-monopole system. Its primary purpose is to illustrate that a comparable binding energy can be obtained from a Schrödinger-type equation when the hydrogen atom is replaced by an analogous proton-monopole system.

The Toy model Hamiltonian is based on the electrostatic interaction between the proton with charge e and mass m_p and a magnetic monopole assumed to have a mass much larger than the proton mass and negative charge (Ze) with Z = -1. These characteristics are not representative of a Dirac monopole. The Toy model binding energy of 25 keV is obtained by the solution of the nonrelativistic Schrödinger equation. Although unphysical, its binding energy coincidentally falls within the range of values summarized in Table 1.

6.2 Pauli Model with Only Realistic Interactions

The second model utilizes the Pauli equation with only realistic interactions. No internal structure effects are included for the proton and no supplemental interactions (e.g., hard cores) are included in the Pauli model. These *ad hoc* effects enhance the proton-monopole binding energy and are noted in the calculations summarized in Table 1 [41-47].

The Pauli equation produces bound states only when the spin-magnetic coupling is sufficiently attractive to counteract the effective centrifugal barrier. Bound states are sensitive to monopole charge, proton spin orientation, and proton spin radial monopole magnetic field coupling strength.

The Pauli equation yields a binding energy of 1.44 keV using only realistic interactions. Higher binding energy values reported in the literature arose from additional phenomenological potentials, which are omitted here to focus on the terms noted in (35). An increased binding energy could result from adding *ad hoc* potentials to the model Hamiltonian. As discussed in Section 2, these additions artificially strengthen the proton-monopole attraction. These additions were utilized in previous calculations to assess additional effects that contribute to the proton-monopole binding energy [41-47].

7.0 Conclusions

Two models are developed to determine the binding energy of a proton-monopole system. One is a conventional Pauli approach, and the other a nonphysical Toy model. The traditional Pauli equation approach only considers realistic spin and monopole effects. No additional *ad hoc* interactions are

considered. The realistic Pauli equation suggests that the proton-monopole system is bound by 1.44 keV.

The unphysical and solely illustrative Toy model yields a binding energy result of 25 keV. Both results are weaker than most previous calculations that included *ad hoc* effects that enhanced the binding energy. The calculated results are sensitive to the assumed Hamiltonian interaction terms.

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